General Constraints on the Propagation of Complex Waves in Closed Lossless Isotropic Waveguides

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***Abstract*—Complex propagation in linear time-invariant lossless isotropic closed waveguides is a theoretically intriguing subject. Complex modes are also practically important in the characterization of discontinuities as they contribute in pairs with complex conjugate (c.c.) propagation constants to local power storage. By the systematic application of Lorentz's reciprocity theorem, we derive the constraints linking complex propagation constant, Poynting's integral, and electromagnetic energy storage per unit length. Previously known conditions are recovered, and novel constraints on the exchange power between the two components of the pairs are derived. It is emphasized that existing relationships individually derived by different methods and scattered in the literature, as well as novel ones, are derived from a single fundamental theorem. This set of constraints is believed to pose the tightest necessary conditions so far for the existence of complex waves.**

***Index Terms*—Closed waveguides, complex modes, electromagnetic theory.**

I. Introduction

COMPLEX modes of lossless reciprocal closed waveguides were first discovered in the numerical simulations of dielectrically loaded cylindrical waveguides in the 1960's [1] and their appearance in inhomogeneous waveguides constituted an intriguing phenomenon ever since [2].

Theoretical investigations on this subject also started in the early 1960's [3]–[5] leading to a set of power and energy constraints and, between the other, highlighting the vanishing of the complex modal Poynting vector of each the two components of the complex mode. This set of constraints was further developed in a simple and elegant manner in order to consider nonbidirectional waveguides [6]; in fact, an important part of the work on complex modes that followed dealt with anisotropic waveguides which are excluded from this paper's investigation.

More recently, complex modes were found in common boxed planar transmission media such as finline [7], [13] and microstrip [8], and were again the object of renewed theoretical attention [9]–[11]. Additional constraints on the fields of complex pairs and their appearance in a shielded circular waveguide were also further described in [12].

Apart from their academic interest, their practical importance also is now well understood: a pair of modes with complex conjugate (c.c.) propagation constants constitutes a reactive mechanism of the guide similar to a real mode below cutoff, and they begin to appear as 19th and 20th mode in a typical 50-Ω microstrip and fin-line [8] on alumina substrate. Consequently, their excitation in the presence of a discontinuity is to be accounted for in order for the equivalent network of the discontinuity to be accurate.

While early studies such as [3]–[6] are based on the direct manipulation of Maxwell's equations, more recent ones like [9]–[11] rely on a discretization of the propagation problem in the guide in terms of the TE/TM modes of the perfectly conducting empty guide. In [13], the hybrid modes at cutoff were taken as basis for the discretization.

A more recent approach to the investigation of the propagation properties of modes in isotropic passive or active closed waveguide was based instead on the systematic use of the Lorentz's reciprocity theorem [14] without recourse to any discrete representation. Although that study was not focused on complex modes in lossless guides, it nonetheless produced an additional relationship between frequency variation of the complex propagation constant and energy storage.

However, each of the two modes constituting the pair with c.c. propagation constants cannot physically exist in absence of the other so that any physical field can only be expressed as a combination of the two.

In [14], the problem was formulated in terms of the properties of an *individual* mode; hence, exchange properties between two coexisting modes forming the pair escaped investigation.

It is the purpose of this paper to systematically apply Lorentz's theorem to the whole *pair*. It will be seen that besides recovering and, in one case, modifying somewhat a constraint that was previously derived, this approach yields additional novel relationships between the complex exchange power, the propagation constant, and the stored energy. To the authors' best knowledge, the whole set of constraints constitutes the tightest necessary conditions so far for the existence of complex modes.

Constraints such as these prove useful in predicting the onset of complex waves, in checking numerical data produced by field analysis programs, and in avoiding spurious solutions.

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Fig. 1. Geometry of the problem.

II. Application of Lorentz's Theorem to Complex Modes

The differential form of Lorentz's theorem for uniform reciprocal waveguides is expressed as

$$\frac{\partial}{\partial z}\iint_S [](\mathbf{E}_t \times \mathbf{H}'_t - \mathbf{E}'_t \times \mathbf{H}_t)\cdot \mathbf{z}\, ds$$
$$= j\omega \iint_S [(\varepsilon-\varepsilon')\mathbf{E}\cdot\mathbf{E}' - (\mu-\mu')\mathbf{H}\cdot\mathbf{H}']\, ds$$
$$+ \iint_S [(\mathbf{J}\cdot\mathbf{E}' - \mathbf{J}'\cdot\mathbf{E}) - (\mathbf{M}\cdot\mathbf{H}' - \mathbf{M}'\cdot\mathbf{H})]\, ds \quad (1)$$

where $\mathbf{E}$, $\mathbf{H}$ is solution consistent with the electric and magnetic currents and permeability distributions ($\mathbf{J}$, $\mathbf{M}$, ε, μ), while ($\mathbf{E}'$, $\mathbf{H}'$) is consistent with ($\mathbf{J}'$, $\mathbf{M}'$, ε', μ'), i.e., satisfying Maxwell's equations

$$\nabla\times\mathbf{E} = -j\,\omega\,\mu\,\mathbf{H} - \mathbf{M}$$
$$\nabla\times\mathbf{H} = j\,\omega\,\varepsilon\,\mathbf{E} + \mathbf{J} \quad (2)$$

together with the appropriate boundary conditions.

In (1), the suffix "t" denotes the transverse part, $\mathbf{z}$ the unit vector in the longitudinal direction, and S the cross section of the guide (see Fig. 1).

In the following, we will first consider sources to be absent, i.e., $\mathbf{J} = \mathbf{J}' = \mathbf{M} = \mathbf{M}' = \mathbf{0}$ and, moreover, we shall always assume ($\mathbf{E}$, $\mathbf{H}$) to correspond to a progressive complex mode, in particular

$$\mathbf{E}(x,y,z) = \mathbf{e}_1(x,y)e^{-\gamma_1 z} + \mathbf{e}_2(x,y)e^{-\gamma_2 z}$$
$$\mathbf{H}(x,y,z) = \mathbf{h}_1(x,y)e^{-\gamma_1 z} + \mathbf{h}_2(x,y)e^{-\gamma_2 z} \quad (3)$$

with

$$\gamma_1 = \gamma_2^* = \alpha + j\beta = \sqrt{\alpha^2+\beta^2}e^{j\varphi}.$$

The fields ($\mathbf{e}_1$, $\mathbf{h}_1$) and ($\mathbf{e}_2$, $\mathbf{h}_2$) form the complex mode pair; although the fields of the pair are not, in general, c.c. of each other, the following relationships are easily derived from [13]

$$\mathbf{e}_{t_1} = \mathbf{e}_{t_2}^* \quad \mathbf{h}_{t_1} = \mathbf{h}_{t_2}^* \quad e_{z_1} = -e_{z_2}^* \quad h_{z_1} = -h_{z_2}^*. \quad (4)$$

It should be noted that we are dealing with isotropic waveguides: for this class, γ^2 and not just γ is the eigenvalue of the telegrapher's equation [13]. Hence, the pairs ($+\gamma$, $+\gamma^*$) and ($-\gamma$, $-\gamma^*$) are independent solutions. Dealing with the quartet ($+\gamma$, $+\gamma^*$, $-\gamma$, $-\gamma^*$) only becomes necessary when the waveguide is girotropic.

At the same time, four different choices of ($\mathbf{E}'$, $\mathbf{H}'$, ε', μ') will be assumed, which are summarized in Table I. It is emphasized that the first two of these correspond to physically realizable situations, whereas the latter two are just formal solutions of Maxwell's equations with boundary conditions without physical counterpart, as the corresponding permittivities are negative.

TABLE I
The Four Canonical Choices of Fields and Sources for the Systematic Application of Lorentz's Reciprocity Theorem

Choice	(E',H',ε',μ')	Direction of propagation
i	($-\mathbf{E}^*$,$\mathbf{H}^*$,ε,μ)	*progressive*
ii	($-\mathbf{E}^*$,$\mathbf{H}^*$,ε,μ)	*regressive*
iii	($\mathbf{E}^*$,$\mathbf{H}^*$,-ε,-μ)	*progressive*
iv	($\mathbf{E}^*$,$\mathbf{H}^*$,-ε,-μ)	*regressive*

i) First Choice: ($\mathbf{E}'$, $\mathbf{H}'$, ε', μ') = ($-\mathbf{E}^*$, $\mathbf{H}^*$, ε, μ): *progressive wave*. In fact, by taking the c.c. of (2), it is evident that i) is an acceptable solution of Maxwell's equation.

After substituting i) in (1), we obtain

$$\frac{\partial}{\partial z}\iint_S \Big\{\Big[(\mathbf{e}_{t_1}\times\mathbf{h}_{t_1}^* + \text{c.c.}) + (\mathbf{e}_{t_2}\times\mathbf{h}_{t_2}^* + \text{c.c.})\Big]e^{-2\alpha z}$$
$$+ \Big[(\mathbf{e}_{t_1}\times\mathbf{h}_{t_2}^* + \mathbf{e}_{t_2}\times\mathbf{h}_{t_1}^*)e^{-2\gamma_1 z} + \text{c.c.}\Big]\Big\}\cdot\mathbf{z}\, ds = 0.$$

By defining

$$P_{ij} \equiv |P_{ij}|e^{j\psi_{ij}} \equiv \iint_S \mathbf{e}_{t_i}\times\mathbf{h}_{t_j}^*\cdot\mathbf{z}ds, \qquad i,j = 1,2$$

with $\text{Re}(P_{ii}) \geq 0$. The above equation reduces to

$$\alpha\{\text{Re}(P_{11}) + \text{Re}(P_{22})\}e^{-2\alpha z}$$
$$+ \tfrac{1}{2}\{(\alpha + j\beta)(P_{12} + P_{21}^*)e^{j2\beta z} + \text{c.c.}\} = 0. \quad (5)$$

Since the z-dependent terms must vanish separately for arbitrary z, (5) yields

$$\text{Re}(P_{11}) = -\text{Re}(P_{22})$$

and, consequently,

$$\text{Re}(P_{11}) = \text{Re}(P_{22}) = 0 \quad (6a)$$

as well as

$$P_{12} = -P_{21}^*. \quad (6b)$$

ii) Second Choice: ($\mathbf{E}'$, $\mathbf{H}'$, ε', μ') = ($-\mathbf{E}^*$, $\mathbf{H}^*$, ε, μ,): *regressive wave*. By substituting ii) in (2), it is seen that this too is acceptable, implying

$$\mathbf{E}'_t(x,y,z) = -\mathbf{e}_{t_1}^*(x,y)e^{\gamma_1^* z} - \mathbf{e}_{t_2}^*(x,y)e^{\gamma_2^* z}$$
$$\mathbf{H}'_t(x,y,z) = -\mathbf{h}_{t_1}^*(x,y)e^{\gamma_1^* z} - \mathbf{h}_{t_2}^*(x,y)e^{\gamma_2^* z}. \quad (7)$$

Substituting in (1) gives

$$\text{Im}(P_{11}) = \text{Im}(P_{22}) = 0 \quad (8)$$

iii) Third Choice: $(\mathbf{E}', \mathbf{H}', \varepsilon', \mu', \mathbf{J}', \mathbf{M}') = (\mathbf{E}^*, \mathbf{H}^*, -\varepsilon, -\mu, 0, 0)$: *forward propagating c.c. wave*, i.e.,

$$\begin{aligned}\mathbf{E}'(x, y, z) &= \mathbf{E}^*(x, y, z)\\ &= e^{-\alpha z}\{\mathbf{e}_1^*(x, y)e^{j\beta z} + \mathbf{e}_2^*(x, y)e^{-j\beta z}\}\\ \mathbf{H}'(x, y, z) &= \mathbf{H}^*(x, y, z)\\ &= e^{-\alpha z}\{\mathbf{h}_1^*(x, y)e^{j\beta z} + \mathbf{h}_2^*(x, y)e^{-j\beta z}\}. \quad (9)\end{aligned}$$

Substituting in (1) produces

$$\begin{aligned}&-2e^{-2\alpha z}\{\alpha \mathrm{Im}(P_{11} + P_{22})\\ &\quad + 2\sqrt{\alpha^2+\beta^2}|P_{12}|\sin(\varphi + \psi_{12} - 2\beta z)\}\\ &= 2\omega \iint_S \left(\varepsilon\frac{|\mathbf{E}|^2}{2} - \mu_0\frac{|\mathbf{H}|^2}{2}\right) dS \equiv 2L. \quad (10)\end{aligned}$$

It is noted that the right-hand side (RHS) contains the angular frequency ω times the difference of the stored energies in the electric and magnetic fields per unit length, i.e., the stored power (Lagrangian function L).

We now rewrite in the RHS

$$\begin{aligned}|\mathbf{E}|^2 &= e^{-2\alpha z}\{|\mathbf{e}_1|^2 + |\mathbf{e}_2|^2 + (\mathbf{e}_1\cdot\mathbf{e}_2 e^{-2j\beta z} + \text{c.c.})\}\\ &\equiv e^{-2\alpha z}\{|\mathbf{e}_1|^2 + |\mathbf{e}_2|^2 + 2|\mathbf{e}_1\cdot\mathbf{e}_2|\cos(\chi_e - 2\beta z)\}|\mathbf{H}|^2\\ &\equiv e^{-2\alpha z}\{|\mathbf{h}_1|^2 + |\mathbf{h}_2|^2 + 2|\mathbf{h}_1\cdot\mathbf{h}_2|\cos(\chi_h - 2\beta z)\}\end{aligned} \quad (11)$$

where the phases $\chi_{e,h}$ are defined in the following. We introduce, moreover, the quantities below with dimensions of energy densities

$$\begin{aligned}E_{e1} &= \iint_S \frac{\varepsilon}{2}|\mathbf{e}_1|^2\,dS;\; E_{e2} = \iint_S \frac{\varepsilon}{2}|\mathbf{e}_2|^2\,dS\\ \tilde{E}_{e12} &= \iint_S \frac{\varepsilon}{2}\mathbf{e}_1\mathbf{e}_2^*\,dS = e^{j\chi_e}E_{e12}, \qquad E_{e12}\ \text{real}\\ E_{h1} &= \iint_S \frac{\mu_0}{2}|\mathbf{h}_1|^2\,dS, \text{etc.,}\end{aligned}$$

and

$$L_1 = \omega(E_{e1} - E_{h1}), L_2 = \omega(E_{e2} - E_{h2})$$

with $E_{e1} = E_{e2}$, $E_{h1} = E_{h2}$, and $L_1 = L_2$ on account of (4).

Using (8), we recast condition (10) as

$$\begin{aligned}&-2\sqrt{\alpha^2+\beta^2}|P_{12}|\{\sin(\varphi+\psi_{12})\cos(2\beta z)\\ &\qquad - \cos(\varphi+\psi_{12})\sin(2\beta z)\}\\ &= L_1 + L_2 + 2\omega\{E_{e_{12}}[\cos(\chi_e)\cos(2\beta z)\\ &\qquad + \sin(\chi_e)\sin(2\beta z)]\}\\ &\quad - E_{h_{12}}[\cos(\chi_h)\cos(2\beta z) + \sin(\chi_h)\sin(2\beta z)]. \quad (12)\end{aligned}$$

Since the above must be satisfied for arbitrary z, we deduce

$$L_1 + L_2 = 0 \quad \text{or} \quad E_{e1} + E_{e2} = E_{h1} + E_{h2}. \quad (13a)$$

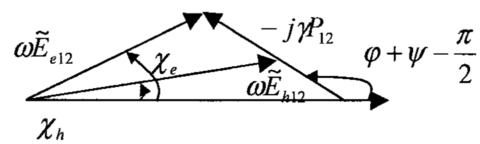

Fig. 2. Geometrical interpretation of (14a) and (14b).

where, moreover, $E_{e1} = E_{e2}$, $E_{h1} = E_{h2}$ owing to (4). After setting for brevity $\psi_{12} = \psi$, we obtain

$$\begin{aligned}&-\sqrt{\alpha^2+\beta^2}|P_{12}|\sin(\varphi+\psi)\\ &\qquad = \omega[E_{e_{12}}\cos(\chi_e) - E_{h_{12}}\cos(\chi_h)]\\ &\sqrt{\alpha^2+\beta^2}|P_{12}|\cos(\varphi+\psi)\\ &\qquad = \omega[E_{e_{12}}\sin(\chi_e) - E_{h_{12}}\sin(\chi_h)]. \quad (13b)\end{aligned}$$

The latter (13b) can be written as

$$-\cot(\varphi+\psi) = \frac{E_{e_{12}}\sin(\chi_e) - E_{h_{12}}\sin(\chi_h)}{E_{e_{12}}\cos(\chi_e) - E_{h_{12}}\cos(\chi_h)}. \quad (14a)$$

$$|\gamma P_{12}|^2 = \omega^2\{E_{e_{12}}^2 + E_{h_{12}}^2 - 2E_{e_{12}}E_{h_{12}}\cos(\chi_e - \chi_h)\}. \quad (14b)$$

In the complex plane, (14a) and (14b) afford the geometrical interpretation of Fig. 2 and can be written compactly as the following single complex equation:

$$\gamma P_{12} = j\omega(\tilde{E}_{e_{12}} - \tilde{E}_{h_{12}}). \quad (14c)$$

iv) Fourth Choice: $(\mathbf{E}', \mathbf{H}', \varepsilon', \mu') = (\mathbf{E}^*, \mathbf{H}^*, -\varepsilon, -\mu)$: *regressive wave*. Again, this is an acceptable choice upon consideration of (2).

Hence, we assume

$$\begin{aligned}\mathbf{E}'(x, y, z) &= e^{\alpha z}\{\mathbf{e}_{t_1}^*(x, y)e^{-j\beta z} + \mathbf{e}_{t_2}^*(x, y)e^{+j\beta z}\\ &\quad - \mathbf{z}[e_{z_1}^*(x, y)e^{-j\beta z} + e_{z_2}^*(x, y)e^{+j\beta z}]\}\end{aligned} \quad (15a)$$

$$\begin{aligned}\mathbf{H}'(x, y, z) &= e^{\alpha z}\{-\mathbf{h}_{t_1}^*(x, y)e^{-j\beta z} - \mathbf{h}_{t_2}^*(x, y)e^{j\beta z}\\ &\quad + \mathbf{z}[h_{z_1}^*(x, y)e^{-j\beta z} + h_{z_2}^*(x, y)e^{+j\beta z}]\}.\end{aligned} \quad (15b)$$

With this last choice, the left-hand side (LHS) of (1) vanishes and we are left with

$$\begin{aligned}&\iint_S \frac{\varepsilon}{2}\{(|\mathbf{e}_{t_1}|^2 - |\mathbf{e}_{z_1}|^2)e^{-2j\beta z} + (\mathbf{e}_{t_1}\cdot\mathbf{e}_{t_2}^* + \text{c.c.})\\ &\qquad - (e_{z_1}\cdot e_{z_2}^* + \text{c.c.}) + (|\mathbf{e}_{t_2}|^2 - |e_{z_2}|^2)e^{2j\beta z}\}dS\\ &+ \iint_S \frac{\mu_0}{2}\{(|\mathbf{h}_{t_1}|^2 - |\mathbf{h}_{z_1}|^2)e^{-2j\beta z} + (\mathbf{h}_{t_1}\cdot\mathbf{h}_{t_2}^* + \text{c.c.})\\ &\qquad - (h_{z_1}\cdot h_{z_2}^* + \text{c.c.}) + (|\mathbf{h}_{t_2}|^2 - |h_{z_2}|^2)e^{2j\beta z}\}dS = 0\end{aligned} \quad (16)$$

again the vanishing of the independent z-variations for arbitrary z leads to the following two conditions:

$$E_{e_{t12}}\cos\chi_{e_t} + E_{h_{t12}}\cos\chi_{h_t} = E_{e_{z12}}\cos\chi_{e_z} + E_{h_{z12}}\cos\chi_{e_z} \quad (17a)$$

where the suffixes t, z denote, respectively, the transverse and longitudinal parts, the suffixes e, h the electric and magnetic parts of the relevant quantities, and

$$E_{t1} = E_{z1} \quad E_{t2} = E_{z2} \tag{17b}$$

E_{t1} denotes the total electromagnetic energy density of the transverse part of the first component ($i = 1$) of the pair, E_{z1} the corresponding density of the longitudinal part, and similarly for the second component of the pair ($i = 2$). Moreover, owing to (4), $E_{e1} = E_{e2}$, $E_{h1} = E_{h2}$.

The above four choices are summarized in Table I, whereas the results are combined in the following theorem:

Theorem: The conditions necessary for the existence of complex modes in a linear time-invariant lossless isotropic waveguide are given by

i) $P_{11} = P_{22} = 0$;
ii) $P_{12} = -P_{21}^*$;
iii) $\gamma P_{12} = j\omega(\tilde{E}_{e_{12}} - \tilde{E}_{h_{12}})$;
iv) $\mathrm{Re}(\tilde{E}_{t_{12}}) = \mathrm{Re}(\tilde{E}_{z_{12}})$;
v) $E_{t_1} = E_{z_1} = E_{t_2} = E_{z_2}$;
vi) $E_e = E_{e_1} + E_{e_2} = E_{h_1} + E_{h_2} = E_h$.

In particular, condition i) implying the vanishing of the integrals of the Poynting vectors of each of the two components of the complex wave is well known [1]–[6]. Condition v), also holding for mode below cutoff, was obtained in [6], together with relationship vi), requiring the balance between electric and magnetic energies of the whole complex wave. Condition ii), stating that the "exchange powers" between 1 and 2 and vice versa are the c.c. and opposite of each other, was first derived in [10]; conditions iii) and iv), however, are quite new: iii) expresses the fact that the product of complex propagation constant and integral of the "exchange" power is proportional to the difference between electric and magnetic complex "exchange" energies, and iv) states that transverse and longitudinal complex exchange energies have identical real parts.

It is, however, emphasized that the four existing relationships, individually derived by different methods in different references, as well as the two novel ones, are derived here just by means of the systematic application of Lorentz's reciprocity theorem (1).

III. Frequency Variation

Neglecting material dispersion, differentiation of the source-free Maxwell (1) with respect to frequency gives

$$\nabla \times \frac{\partial \mathbf{E}}{\partial \omega} = -j\omega \frac{\partial \mathbf{H}}{\partial \omega} - j\mu \mathbf{H}$$
$$\nabla \times \frac{\partial \mathbf{H}}{\partial \omega} = j\omega \frac{\partial \mathbf{E}}{\partial \omega} + j\varepsilon \mathbf{E} \tag{18}$$

which can again be interpreted as Maxwell equations for the fields

$$\left(\frac{\partial \mathbf{E}}{\partial \omega}, \frac{\partial \mathbf{H}}{\partial \omega}\right)$$

satisfying the same boundary conditions of the original fields in presence of a magnetic current source $\mathbf{M} = -j\mu\mathbf{H}$ and an electric current source $\mathbf{J} = -j\varepsilon\mathbf{E}$.

Again, systematic application of Lorentz's theorem to the pair of complex modes produces several novel relationships between the frequency derivatives of γ, P, and stored energies. These are not reported here for sake of brevity, as their derivation is tedious and their physical interpretation mostly obscure, except for the one result reported below omitting proof, namely,

$$\frac{\partial}{\partial \omega}[P_{11}(\omega) + P_{22}(\omega)] = 0. \tag{19}$$

Equation (19) predicts the existence of a complex mode over an open interval of frequencies rather than just locally at a point: the latter "broad-band" property was observed in [13], while investigating a numerical example of fin-line.

IV. Conclusions

Systematic application of Lorentz's theorem provides the means of generating the set of tightest necessary conditions so far for the existence of complex modes in closed lossless reciprocal waveguides, quite independently of any particular discrete representation of the fields.

Such conditions can be exploited for predicting the likely excitation of complex waves as well as for checking and directing the numerical solution of the field problem, in particular in avoiding spurious roots.

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